# EE 232 Lightwave Devices
# Lecture 10: Intersubband Absorption in Quantum Wells


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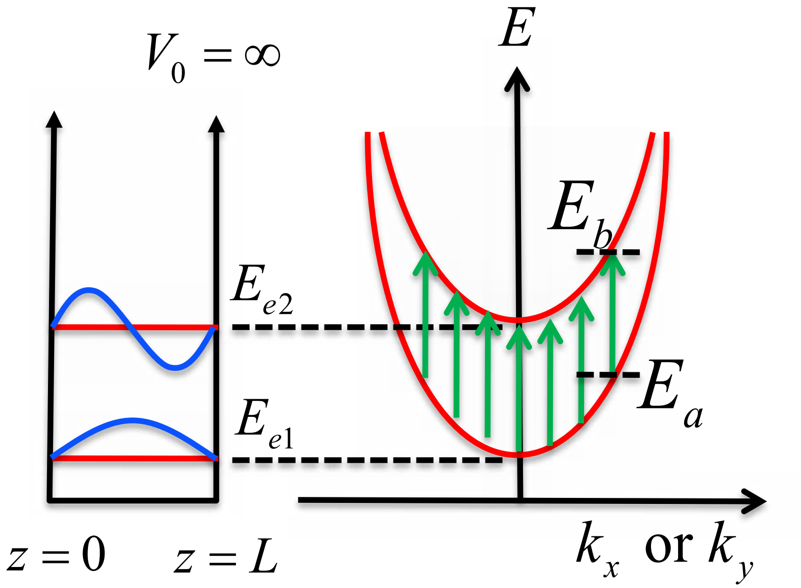
# Intersubband Transition in Quantum Wells



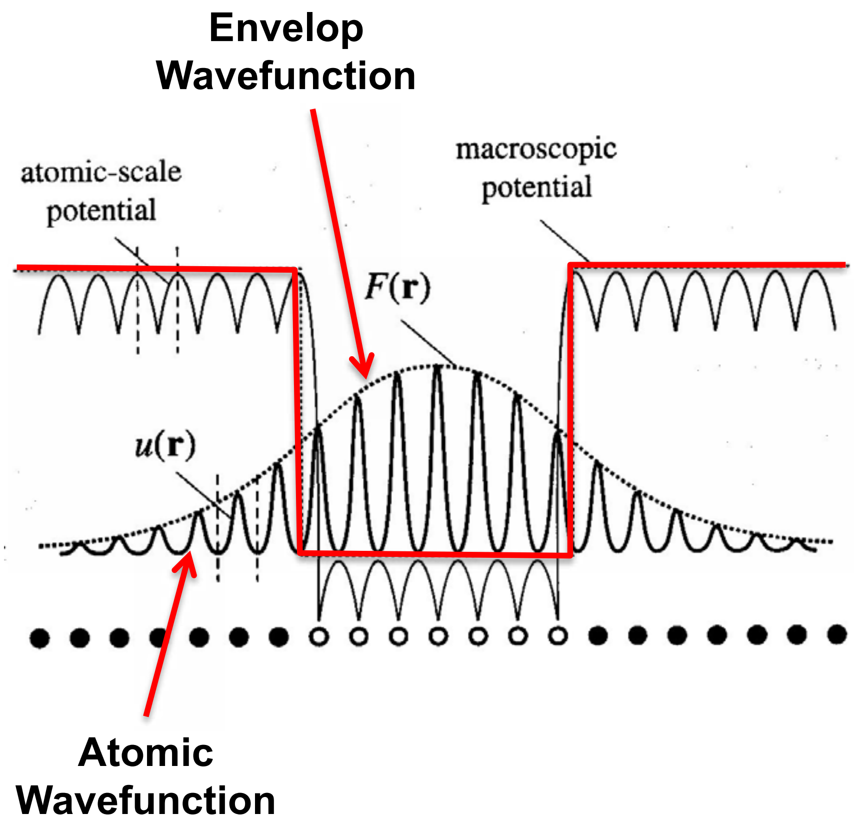

For a 10-nm QW in GaAs

$$E_{e1}^{e2} = E_{e2} - E_{e1}$$
$$= 224 \text{ meV} - 56 \text{ meV}$$
$$= 168 \text{ meV}$$
$$\lambda = \frac{1.24}{0.168} = 7.4 \mu m$$

- **Transition between quantized energy levels in a quantum well by absorption or emission of a photon**
- **Absorption**
  - **Infrared photodetectors**
  - **Thremal imager: blackbody radiation of human body ~ 10 μm**
  - **3 ~ 5 and 8 ~ 10 μm wavelength bands particularly interesting**
- **Emission**
  - **Gain media for quantum cascaded lasers (QCL)**
  - **Long wavelength emission coincides with molecular vibration spectra**

# Optical Matrix Element for Intersubband Transition

## Quantum Well Wavefunction




$$|a\rangle = \psi_a(\vec{r}) = u_c(\vec{r})\frac{e^{i\vec{k}_t\cdot\vec{\rho}}}{\sqrt{A}}\phi_1(z)$$

$$|b\rangle = \psi_b(\vec{r}) = u_c(\vec{r})\frac{e^{i\vec{k}_{t'}\cdot\vec{\rho}}}{\sqrt{A}}\phi_2(z)$$

$$H'_{ba} = -\vec{E}\cdot\vec{\mu}_{ba}$$

$$\vec{\mu}_{ba} = \langle b|e\vec{r}|a\rangle$$

$$\approx \langle u_c(\vec{r})|u_c(\vec{r})\rangle \int \frac{e^{i\vec{k}_{t'}\cdot\vec{\rho}}}{\sqrt{A}}\frac{e^{i\vec{k}_t\cdot\vec{\rho}}}{\sqrt{A}}d\vec{\rho}$$

$$\cdot\int \phi_2^*(z)e\vec{r}\phi_1(z)d\vec{r}$$

**Slowly Varying Envelop Approx**

$$= 1\cdot\delta_{\vec{k}_t,\vec{k}_{t'}}\mu_{21}\hat{z}$$

# Absorption Coefficient for Intersubband Transition

$$\alpha(\hbar\omega) = \frac{\pi\omega}{n_r c\varepsilon_0} \cdot \frac{2}{V} \sum_{\vec{k}_t} g(E_b - E_a - \hbar\omega) \left|\hat{e} \cdot \vec{\mu}_{ba}\right|^2 \left(f_a - f_b\right)$$

The summation is over all electron states: $\vec{k}_t = k_x \hat{x} + k_y \hat{y}$

We need to consider the finite width of the energy spread

(otherwise the absorption is a delta function with infinite absorption peak)

$$g(\Delta E) = \frac{1}{\pi} \frac{\Gamma/2}{\Delta E^2 + \left(\Gamma/2\right)^2} \quad \text{(Lorentzian Lineshape)}$$

$\left|\hat{e} \cdot \vec{\mu}_{ba}\right|^2 = \left|\mu_{21}\right|^2$ only when $\hat{e} = \hat{z}$ (TM polarization)

$E_b - E_a = E_{e1}^{e2}$ , independent of $\vec{k}_t$

$$\alpha(\hbar\omega) = \frac{\pi\omega}{n_r c\varepsilon_0} g(E_{e1}^{e2} - \hbar\omega) \left|\mu_{21}\right|^2 \frac{2}{V} \sum_{\vec{k}_t} \left(f_a - f_b\right)$$

$$\boxed{\alpha(\hbar\omega) = \frac{\pi\omega}{n_r c\varepsilon_0} g(E_{e1}^{e2} - \hbar\omega) \left|\mu_{21}\right|^2 (N_1 - N_2)}$$



Prof. Ming Wu

# Absorption Coefficient for Intersubband Transition

$$\alpha(\hbar\omega) = \frac{\pi\omega}{n_r c\varepsilon_0} g(E_{e1}^{e2} - \hbar\omega)\left|\mu_{21}\right|^2 (N_1 - N_2)$$

(1) $E_1 < F < E_2$

$N_2 \approx 0$

$$\alpha(\hbar\omega) = \frac{\pi\omega}{n_r c\varepsilon_0} g(E_{e1}^{e2} - \hbar\omega)\left|\mu_{21}\right|^2 N_1$$

is proportional to doping concentration

(2) $E_2 < F$

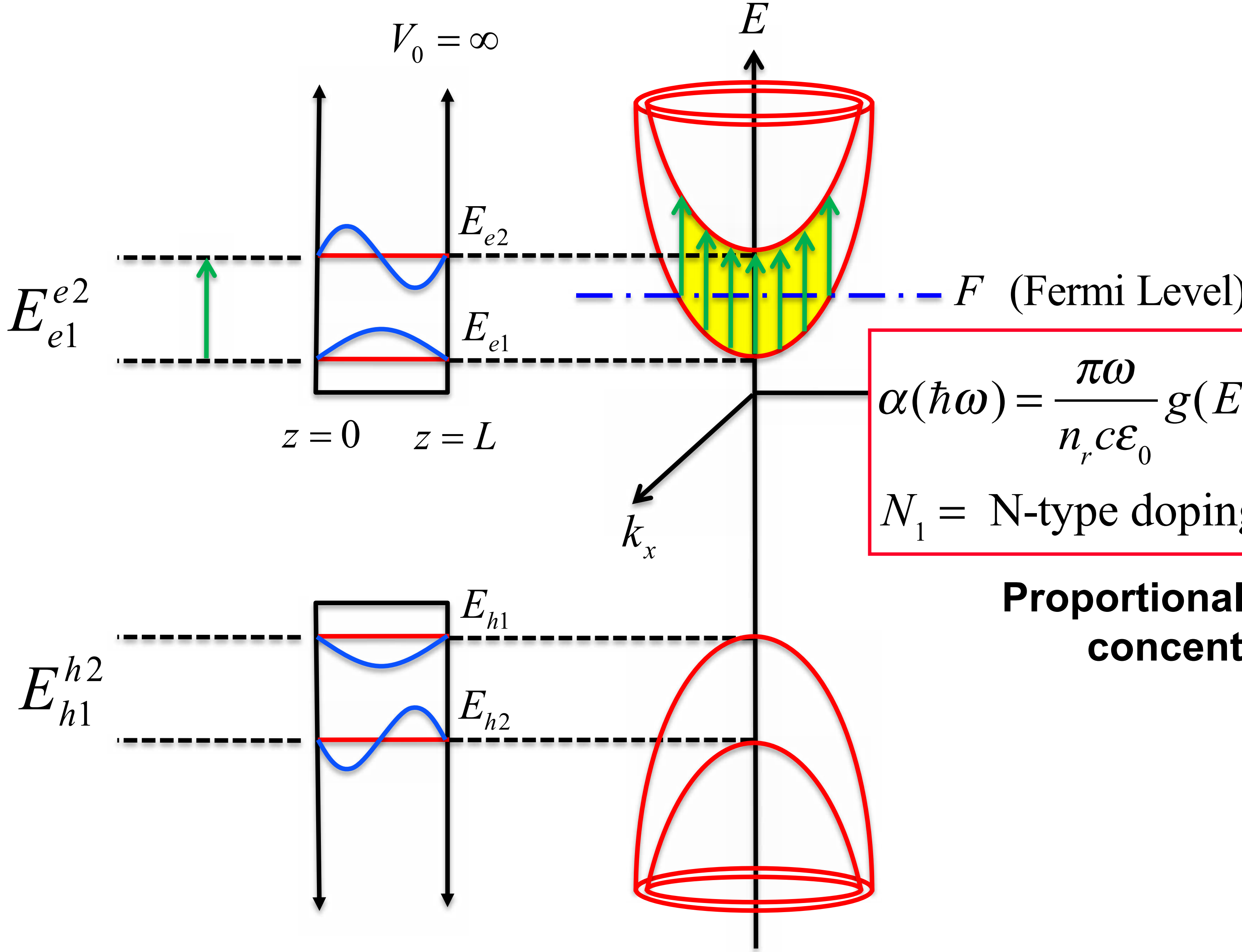
$$N_1 = \frac{m_e^* k_B T}{\pi\hbar^2 L_z} \ln(1 + e^{\frac{F-E_1}{k_B T}}) \approx \frac{m_e^* k_B T}{\pi\hbar^2 L_z} \frac{F - E_1}{k_B T} = \frac{m_e^*}{\pi\hbar^2 L_z}(F - E_1)$$

$$N_2 \approx \frac{m_e^*}{\pi\hbar^2 L_z}(F - E_2)$$

$$\alpha(\hbar\omega) = \frac{\pi\omega}{n_r c\varepsilon_0} g(E_{e1}^{e2} - \hbar\omega)\left|\mu_{21}\right|^2 \frac{m_e^*}{\pi\hbar^2 L_z} E_{e1}^{e2}$$

is a constant, independent of doping concentration

# Intersubband Transition in P-Doped QW (1): Fermi-Level Below Second Subband

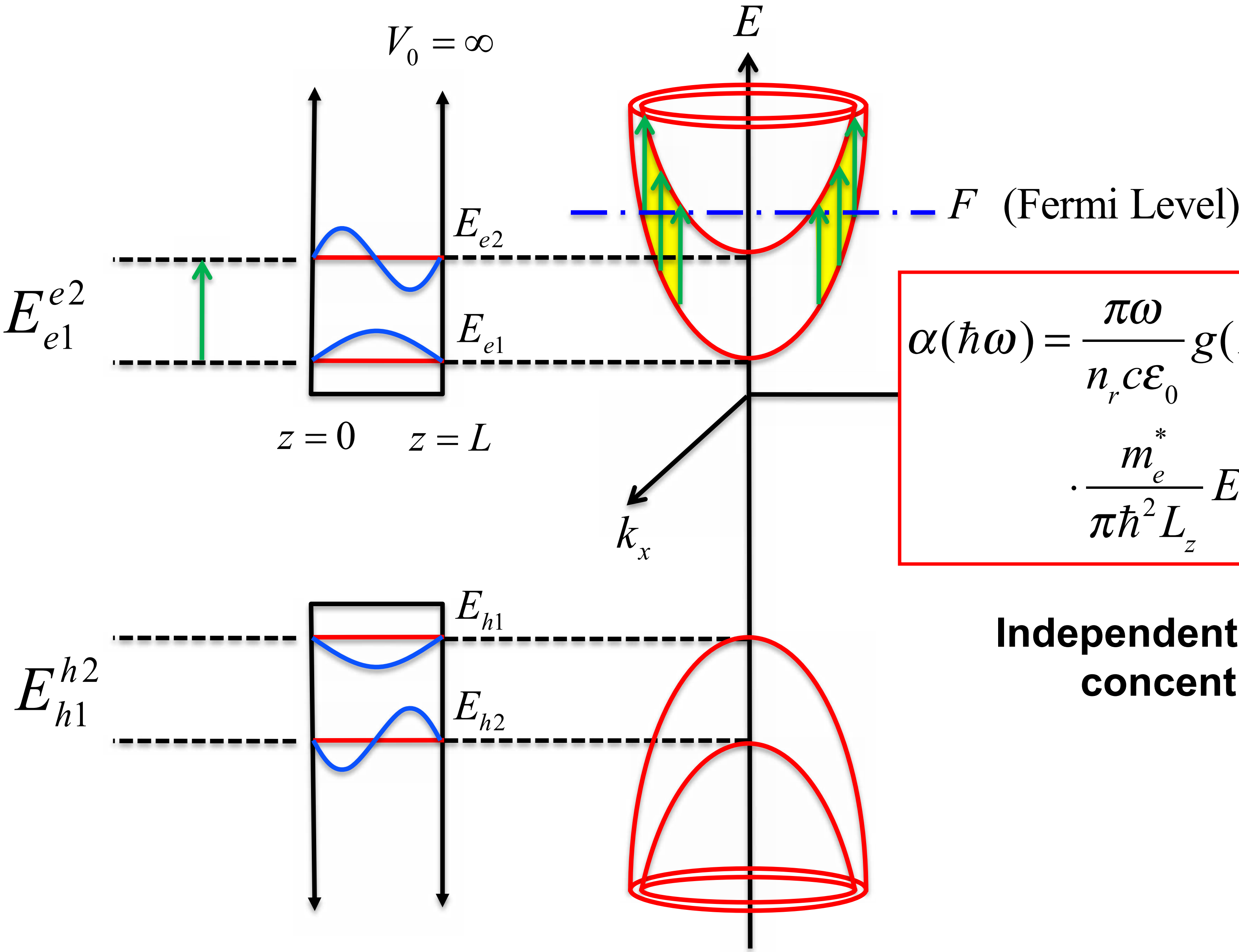



$$\alpha(\hbar\omega) = \frac{\pi\omega}{n_r c\varepsilon_0} g(E_{e1}^{e2} - \hbar\omega)\left|\mu_{21}\right|^2 N_1$$

$N_1 =$ N-type doping concentration

**Proportional to doping concentration**

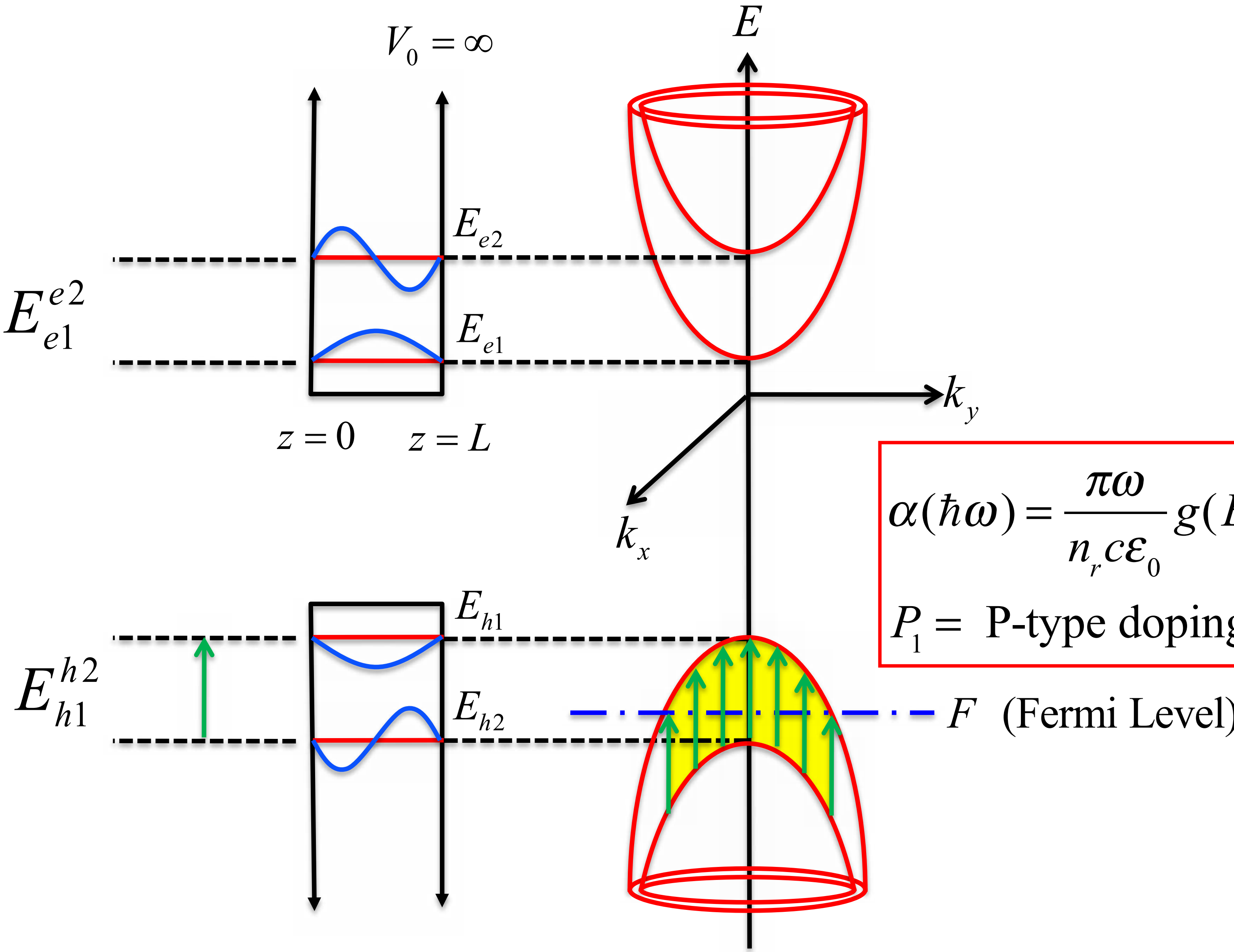
# Intersubband Transition in N-Doped QW (2): Fermi-Level Above Second Subband




$$\alpha(\hbar\omega) = \frac{\pi\omega}{n_r c\varepsilon_0} g(E_{e1}^{e2} - \hbar\omega)\left|\mu_{21}\right|^2 \cdot \frac{m_e^*}{\pi\hbar^2 L_z} E_{e1}^{e2}$$

**Independent of doping concentration**

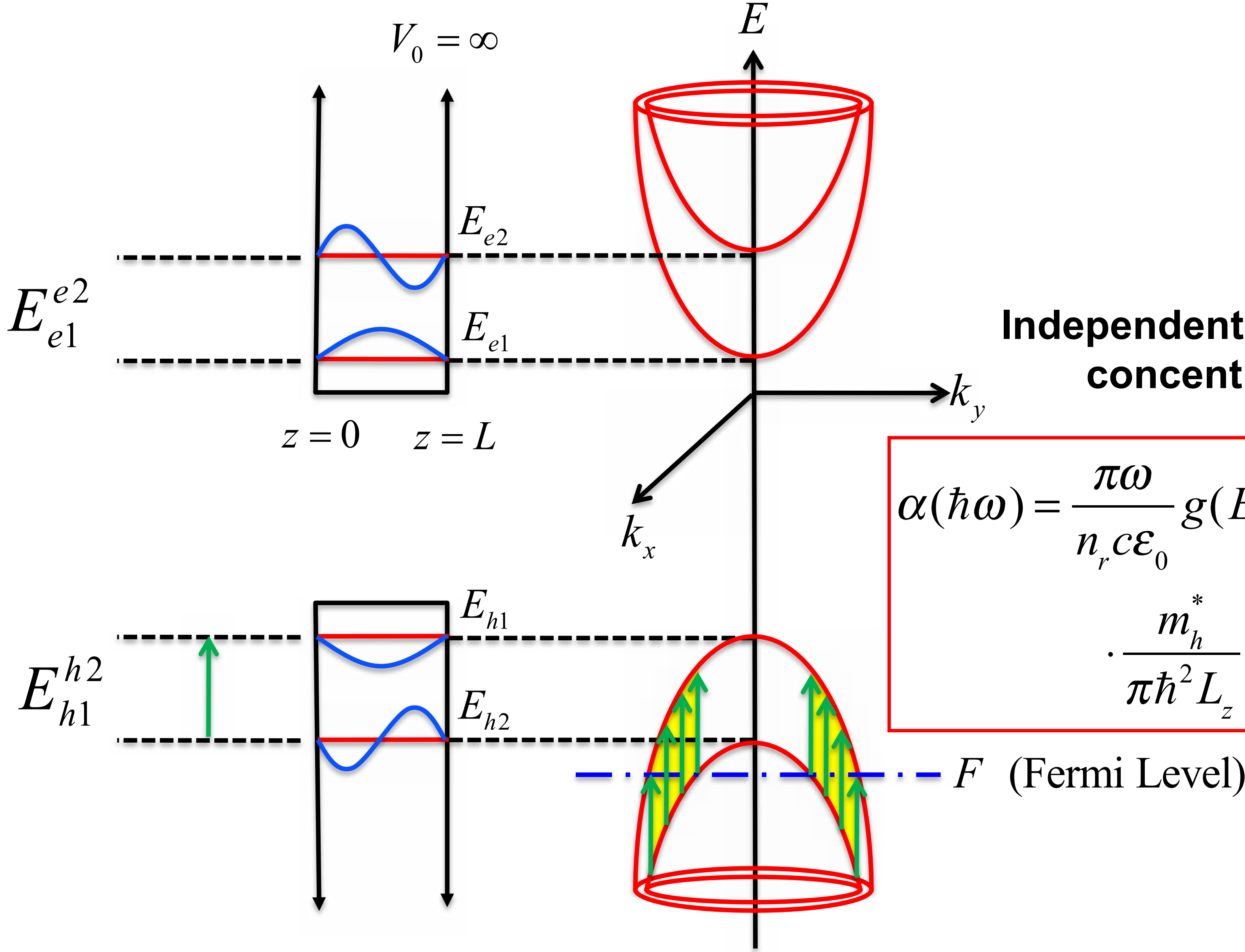
# Intersubband Transition in P-Doped QW (1): Fermi-Level Above Second Subband




$$\alpha(\hbar\omega) = \frac{\pi\omega}{n_r c \varepsilon_0} g(E_{h2}^{h1} - \hbar\omega) \left|\mu_{21}\right|^2 P_1$$

$P_1 =$ P-type doping concentration

# Intersubband Transition in P-Doped QW (2): Fermi-Level Below Second Subband




**Independent of doping concentration**

$$\alpha(\hbar\omega) = \frac{\pi\omega}{n_r c\varepsilon_0} g(E_{h2}^{h1} - \hbar\omega)\left|\mu_{21}\right|^2 \cdot \frac{m_h^*}{\pi\hbar^2 L_z} E_{h2}^{h1}$$

# Intersubband Dipole Moment

$$\phi_1(z) = \sqrt{\frac{2}{L_z}} \sin\left(\frac{\pi}{L_z} z\right)$$

$$\phi_2(z) = \sqrt{\frac{2}{L_z}} \sin\left(\frac{2\pi}{L_z} z\right)$$

$$\mu_{21} = e\int_0^{L_z} \phi_2(z)\cdot z \cdot \phi_1(z) dz$$

$$= \frac{2e}{L_z}\int_0^{L_z} \sin\left(\frac{\pi}{L_z} z\right)\cdot z \cdot \sin\left(\frac{2\pi}{L_z} z\right) dz$$

$$= -\frac{16}{9\pi^2} eL_z$$

Compare with dipole moment of interband transition:

$$\frac{\left|\mu_{21}^{intra}\right|}{\left|\mu_{cv}^{inter}\right|} = \frac{\frac{16}{9\pi^2} eL_z}{er_{cv}} \approx \frac{0.2L_z}{0.4 \text{ nm}}$$

# Example

$L_z = 10nm$

$$E_{e1} = \frac{\hbar^2}{2m_e^*}\left(\frac{\pi}{L_z}\right)^2 = 56 \text{ meV}$$

$$E_{e2} = \frac{\hbar^2}{2m_e^*}\left(\frac{2\pi}{L_z}\right)^2 = 224 \text{ meV}$$

$E_{e1}^{e2} = 168 \text{ meV}$

$\text{N} = 10^{18} \text{ cm}^{-3}$

First, determine if the second subband is occupied.

Find $N_{1,\max}$ , the electron concentration in the first subband

when the Fermi level is at the bottom of the second subband:

$$N_{1,\max} = \frac{m_e^*}{\pi\hbar^2 L_z} E_{e1}^{e2} = 4.7\times10^{18} \text{ cm}^{-3} > \text{N} = 10^{18} \text{ cm}^{-3}$$

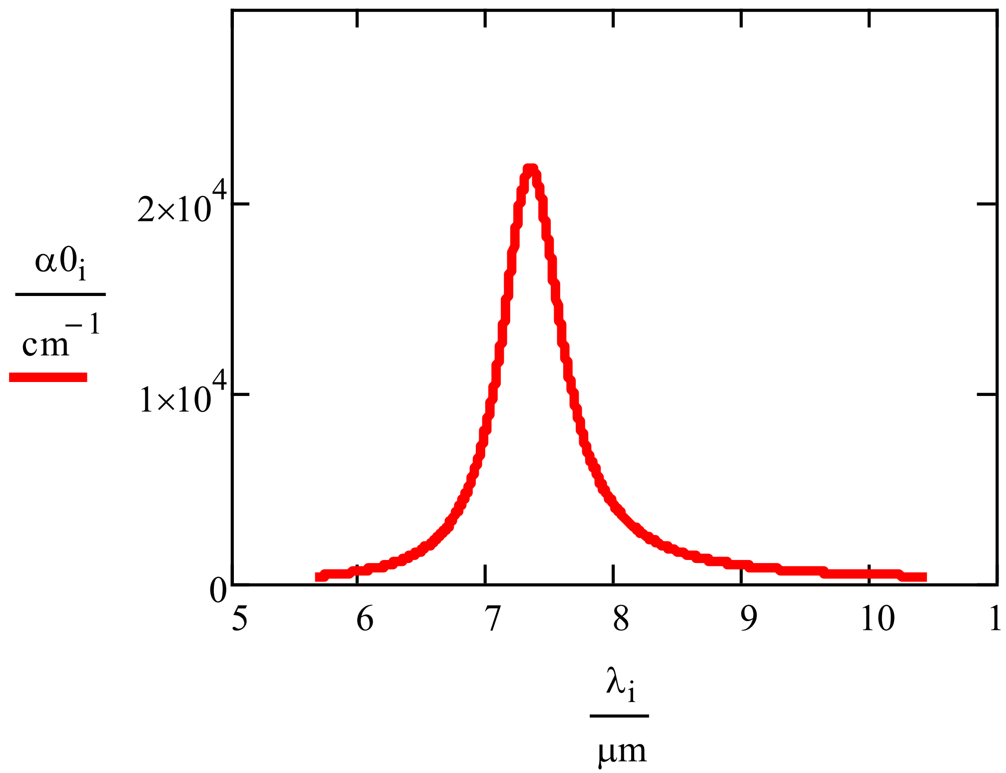
So the second subband is not occupied. $\text{N}_2 = 0$

# Absorption Spectra

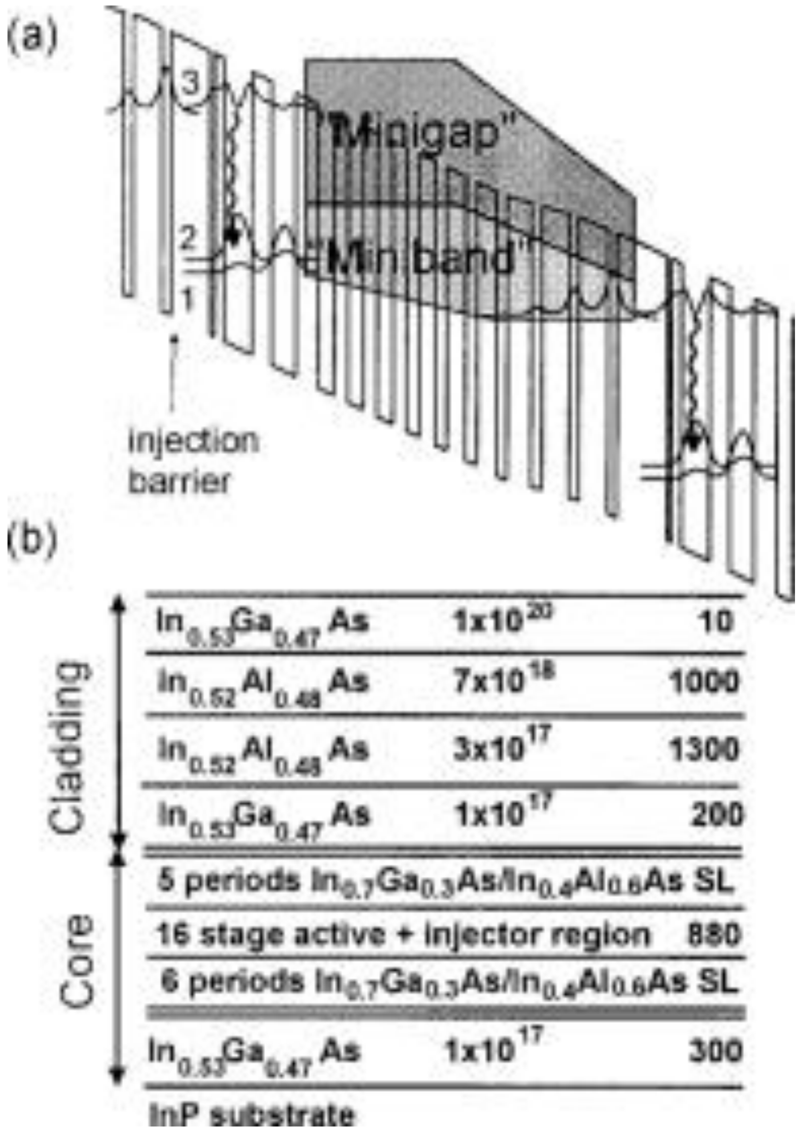
$$\alpha(\hbar\omega)=\frac{\pi\omega}{n_r c\varepsilon_0} g(E_{e1}^{e2}-\hbar\omega)\left(\frac{16}{9\pi^2}eL_z\right)^2 N$$

Peak absorption

$$\alpha_{\max}=\frac{\pi\omega}{n_r c\varepsilon_0}\frac{1}{\pi\left(\frac{\Gamma}{2}\right)}\left(\frac{16}{9\pi^2}eL_z\right)^2 N \approx 2\times10^4\ \text{cm}^{-1}$$



# Quantum Cascade Laser (QCL)




- **Using intersubband transition to provide gain**
- **From THz to IR**
- **Key design**
  - **Upper state should be aligned with a "mini-gap" to prevent direct tunneling loss of upper state electrons**
  - **Lower state should be aligned with a "mini-band" to quickly remove the lower state population**
- **The electrons can be "reused" by cascading the quantum wells**
- **Emission wavelength tailorable by varying thickness of various layers**